$\bar{v}_0$ = mean die velocity
We = Weber number, Equation (1)

Greek Letters

η = viscosity
λ = characteristic time of the fluid
σ = surface tension

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Manuscript received December 16, 1975; revision received and accepted May 17, 1976.

Mass Transfer From Swarms of Bubbles or Drops With Chemical Reactions in Continuous Phase

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Mass transfer with chemical reactions between the continuous phase and a single bubble or drop moving at a low or high Reynolds number has been investigated previously by Ruckenstein, Dang, and Gill (1971, 1973). They considered the rate determining step of the transfer processes was in the continuous phase. Gal-Or and Yaron (1973) studied physical mass transfer between ensembles of drops or bubbles and the continuous phase.

In many practical systems of interest, mass transfer with chemical reactions usually takes place between swarms of bubbles and continuous phase. Therefore, study and understanding of those systems are of extreme importance.

The objective of the first part of the present paper is to analyze the problem of mass transfer from swarms of bubbles with chemical reactions. The second part of the paper is to investigate experimentally a bubble mass transfer system.

When resistance for mass transfer is considered to be in the continuous phase, one can write the following unsteady convective diffusion equation with first-order chemical reaction for swarms of bubbles moving at low Reynolds number:

$$\frac{\partial c}{\partial t} + V_r \frac{\partial c}{\partial r} + \frac{V_\theta}{r}\frac{\partial c}{\partial \theta} = D\frac{\partial^2 c}{\partial r^2} - kc \tag{1}$$

The initial and boundary conditions are

$$c(o, r, \theta) = c_o \tag{2}$$

$$c(t, \infty, \theta) = c_o e^{-kt} \tag{3}$$

$$c(t, a, \theta) = c^* \tag{4}$$

and the velocity profile for swarms of bubbles has been established by Gal-Or and Yaron (1973) as

$$V_r = \frac{3}{2}\frac{U_s}{A}\left[\left(\frac{r}{a}\right)^2 \Phi^{5/3} - B + A\left(\frac{r}{a}\right)^{-1} - \left(\frac{r}{a}\right)^{-3}\right]\cos\theta \tag{5}$$

$$V_\theta = \frac{3}{2}\frac{U_s}{A}\left[\frac{-1}{2}\left(\frac{r}{a}\right)^{-3} - \frac{A}{2}\left(\frac{r}{a}\right)^{-1} + B - 2\left(\frac{r}{a}\right)^2 \Phi^{5/3}\right]\sin\theta \tag{6}$$

$$1 \leq \frac{r}{a} \leq \Phi^{-1/3}$$

where

$$A = 3 + 2\beta + 2\Phi^{5/3}(1 - \beta),$$
$$B = 2 + 2\beta + \Phi^{5/3}(3 - 2\beta), \quad \beta = \frac{\mu_c}{\mu_d + \gamma} \tag{7}$$

The first step toward solving this set of equations is to introduce some dimensionless variables as

$$\omega = \frac{c}{c^*}, \quad Y = \frac{r - a}{a}, \quad \tau = \frac{tD}{a^2}, \quad R = \frac{ka^2}{D},$$

$$Pe = \frac{3aU_s}{2DA}\beta(1-\Phi^{5/3})$$

With these dimensionless variables, one can transform Equations (1) to (6) to dimensionless forms. These equations can be solved readily by means of similarity transformation and Duhamel's theorem as outlined by Ruckenstein, Dang, and Gill (1971, 1973). Hence, one obtains the concentration distribution in the continuous phase

$$\omega(\tau, Y, \theta) = e^{-R\tau}\frac{\partial}{\partial\tau}\int_o^\tau \left\{ \omega_o + (e^{R\lambda}-\omega_o) \times \operatorname{erfc}\frac{Y\sqrt{Pe}\sin^2\theta}{2\left[\dfrac{1-\tan^2\frac{\theta}{2}\exp(-2Pe\tau)}{1+\tan^2\frac{\theta}{2}\exp(-2Pe\tau)} - \cos\theta - \dfrac{1}{3}\left[\dfrac{1-\tan^2\frac{\theta}{2}\exp(-2Pe\tau)}{1+\tan^2\frac{\theta}{2}\exp(-2Pe\tau)}\right]^3 + \dfrac{1}{3}\cos^3\theta\right]^{1/2}} \right\} d\lambda \tag{8}$$

and the Sherwood number can be evaluated as

$$Sh = \frac{1}{1-\omega_o e^{-R\tau}}\sqrt{\frac{Pe}{\pi}}\,e^{-R\tau}\left\{ (1-\omega_o) \times \int_o^\pi \frac{\sin^3\theta d\theta}{\left[\dfrac{1-\tan^2\frac{\theta}{2}\exp(-2Pe\tau)}{1+\tan^2\frac{\theta}{2}\exp(-2Pe\tau)} - \cos\theta - \dfrac{1}{3}\left[\dfrac{1-\tan^2\frac{\theta}{2}\exp(-2Pe\tau)}{1+\tan^2\frac{\theta}{2}\exp(-2Pe\tau)}\right]^3 + \dfrac{1}{3}\cos^3\theta\right]^{1/2}} \right.$$
$$\left. + Re^{R\tau}\int_o^\tau e^{-Rz}\int_o^\pi \frac{\sin^3\theta d\theta dz}{\left[\dfrac{1-\tan^2\frac{\theta}{2}\exp(-2Pez)}{1+\tan^2\frac{\theta}{2}\exp(-2Pez)} - \cos - \dfrac{1}{3}\left[\dfrac{1-\tan^2\frac{\theta}{2}\exp(-2Pez)}{1+\tan^2\frac{\theta}{2}\exp(-2Pez)}\right]^3 + \dfrac{1}{3}\cos^3\theta\right]^{1/2}} \right. \tag{9}$$

Equation (9) has been evaluated numerically. However, a simpler asymptotic equation for long time expression of Equation (9) is established as follows:

$$Sh = \frac{1}{1-\omega_o e^{-R\tau}}\sqrt{\frac{Pe}{\pi}}\left(2\sqrt{\frac{\pi R}{Pe}}\operatorname{erf}\sqrt{\frac{0.5R}{Pe}} + \frac{4}{\sqrt{3}}e^{-\frac{0.5R}{Pe}} - \frac{4}{\sqrt{3}}\omega_o e^{-R\tau}\right) \quad \text{for} \quad \tau >> \frac{0.5}{Pe} \tag{10}$$

When Φ is less than 0.005, one can obtain similar results of the Sherwood numbers from Equations (9) and (10) or from calculation from single bubble or drop.

Mass transfer coefficient calculated from Equations (9) and (10) can be used for design of absorption, desorption, and extraction equipments. In the next section, we will describe an experiment of absorption of carbon dioxide from air bubbles by dilute potassium carbonate solutions. The purpose of the experiment is to substantiate the design information for the work of Steinberg and Dang (1975) and the analysis discussed above.

EXPERIMENT

Room air is used as a source of carbon dioxide. The experiment is carried out at room temperature. Air is first blown through a filter to remove any possible solid impurities. The air flow rate is controlled by a valve, and the volumetric flow rate is measured from a rotameter. The air flow rate is set at 100 cm^3/min. The absorption column is a glass cylinder of 2.8 cm diameter and 8 cm tall. About 35 to 45 cm^3 of 0.01 N potassium carbonate solution is placed in the cylinder as an absorbent. Clean air is passed through a fritted glass placed in the bottom of the cylinder. Bubble motion and size are observed and measured on pictures taken photographically during the run. A meter stick at the front of the column was photographed along with the bubbles. Average bubble diameter of 0.2 cm is measured in the present experiment. Bubbles are spherical in shape, and the volume fraction of the bubbles is usually small, about 0.01. The amount of carbon dioxide absorbed is measured accurately at different time intervals during the experiment by using a total carbon analyzer. The output of the total carbon analyzer is connected to a recorder which gives the traced results of the experiment. The carbon concentration and hence the concentration of carbon dioxide in the 0.01 N potassium carbonate solution are obtained by comparing the measured peak height in the experiment and the peak height of a known reference carbon solution (100 ppm carbon). The pH of the solution is measured by a pH meter.

RESULTS AND DISCUSSIONS

Some experimental results are shown in Figure 1. Since the concentration of carbon dioxide in air is dilute, about 350 ppm, it takes quite a long time for carbon dioxide to transfer and reach the final steady state value in the bulk liquid phase.

Mechanism of absorption of carbon dioxide in carbonate solution is a combination of diffusion and chemical reaction processes. The reaction mechanism proposed by Danckwerts (1970) is a pseudo first-order reaction with respect to the concentration of carbon dioxide under some conditions. The reaction rate constant for the reaction of carbon difoxide and hydroxyl ion is taken to be $10^4 l$/mole s. The pH of 0.01 N potassium carbonate solution is 9.9. The velocity of rise of the bubble is obtained photographically or calculated by the equation derived for swarm of bubbles moving at low Reynolds number flow.

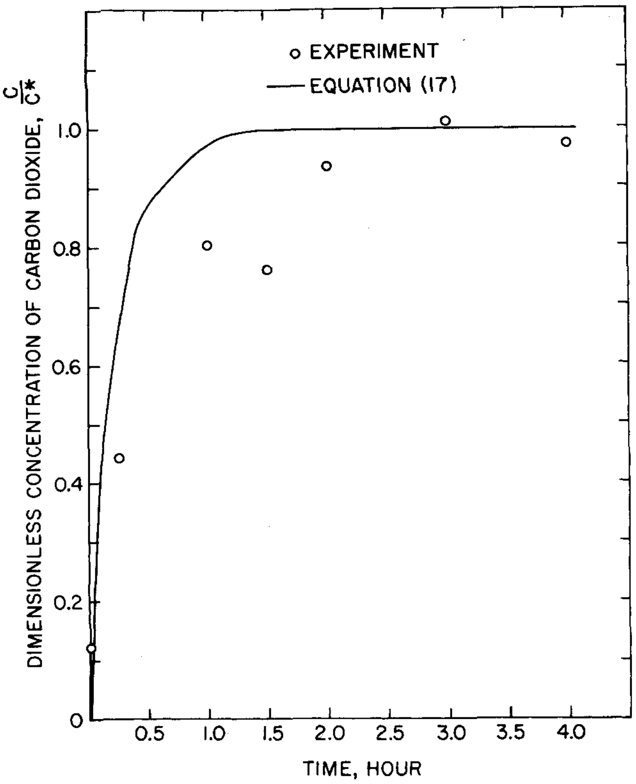

Fig. 1. Mass transfer from swarms of bubbles or drops with chemical reactions in continuous phase.

The macroscopic absorption process can be analyzed by means of a quasi steady state approximation. Since the residence time of the bubbles is much smaller than the time scale of bulk absorption, one can use steady state mass transfer for the swarm of bubbles and unsteady state absorption for the bulk liquid phase. With this assumption, one can obtain the mass balance for the continuous phase as

$$V\frac{dc}{dt} = 4\pi a^2 N_p k_L (c^* - c) \qquad (11)$$

which then gives the bulk concentration of carbon dioxide as

$$\frac{c}{c^*} = 1 - \exp\left(-\frac{3V_b}{aV}k_L t\right) \qquad (12)$$

It is necessary to note here that the time scale in Equation (12) is the time of absorption in the continuous phase, while the time scale in Equation (1) is the residence time of absorption from the bubble. Equation (12) is also shown in Figure 1. In application of Equation (12), the mass transfer coefficient k_L is obtained from Equation (9) or (10) by taking $\tau \rightarrow \infty$. Although Equation (12), which is based on the quasi steady state approximation, is a simple one, it can be used to describe a system well, as can be seen in Figure 1. The present analysis can be applied to other systems such as oxidation by oxygen or ozone and hydrogenation of hydrocarbons.

ACKNOWLEGMENT

The authors wish to thank Jacob Pruzansky for carrying out the experiment.

NOTATION

a = radius of bubble
c = concentration of the continuous phase
c^* = equilibrium concentration of the continuous phase
c_o = initial concentration of the continuous phase
D = molecular diffusivity in the continuous phase
k = reaction rate constant in the continuous phase
k_L = average mass transfer coefficient
N_p = total number of bubbles in the absorption column
U = translational velocity of bubble
V = volume of continuous phase
V_b = volume of bubbles

Greek Letters

β = viscosity ratio defined by Equation (7)
γ = interfacial retardation viscosity
μ_c = viscosity of continuous phase
μ_d = viscosity of dispersed phase
ω = c/c^*
Φ = volume fraction of dispersed phase

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Manuscript received January 30, 1976; revision received and accepted March 8, 1976.

Thermal Conductivity of Liquid Mixtures

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Several methods have been proposed to estimate the thermal conductivity of liquid mixtures; most require empirical parameters or some mixture data (Jamieson and Hastings, 1969). Critical reviews of these methods are available (Reid et al., 1976; Tsederberg, 1965).

In developing the proposed method, the following as-